

Absorption cross sections of real photons at superhigh energies

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A brief review of modern theoretical models describing the process of the absorption of real photons by nucleons at superhigh energies is given. The main aim of the work is an estimation of the theoretical uncertainty of the cross section prediction at photon energies around $10^{19} - 10^{20}$ eV.

1. Introduction

Photons of energy $10^{19} - 10^{20}$ eV are interesting in connection with the problem of the origin of high energy cosmic rays.

The rather abundant fluxes of ultra-high energy (UHE) photons are predicted in top-down models. UHE photons appear also as a result of interactions of UHE cosmic rays (UHECR) with relic radiation.

Search for UHE photons use the comparison of extensive air shower data with results of detailed simulations based on assumptions on the photonuclear cross section.

An extrapolation of low energy data on $\sigma_{\gamma p}$ has been given by the parametrization [1]

$$\sigma_{\gamma p} = 114.3 + 1.647 \ln^2[s/s_0] \quad (\mu\text{b}), \quad (1)$$

where $s_0 = 88.243 \text{ GeV}^2$. The $\ln^2 s$ -law in this formula has been chosen assuming the validity of the vector dominance model (VDM) and additive quark model (see below, Eq. (9)).

More recently, Block and Halzen [2] suggested the extrapolation of $\sigma_{\gamma p}$ using the analyticity arguments. They wrote, as a starting point (ν is the photon energy in the lab frame)

$$\sigma_{\gamma p} = C_0 + C_1 \ln \frac{\nu}{m_p} + C_2 \ln^2 \frac{\nu}{m_p} + \beta \left(\frac{\nu}{m_p} \right)^{\mu-1}. \quad (2)$$

Constants C_i are constrained using the precise low energy fit at $\sqrt{s} < 2.0 \text{ GeV}$ (given by Damashek and Gilman [3]). The authors showed also that the fit with $C_2 = 0$ (i.e., the $\ln s$ -dependence in asymptotic) is not good (from a point of view of the χ^2 -analysis).

2. Eikonal (minijet) models

The total photoabsorption cross section in eikonalized minijet models is calculated by the basic formula (see, e.g., [4])

$$\sigma_{\gamma p}(s) = 2 \frac{\alpha_{em}}{\pi} n_f \langle e^2 \rangle \int d^2b \times \int_{(k_{\perp 0}^{min})^2} \frac{dk_{\perp 0}^2}{k_{\perp 0}^2} [1 - e^{-\chi(s,b,k_{\perp 0}^2)}]. \quad (3)$$

Here, $\chi(s, b, k_{\perp 0}^2)$ is the factorised eikonal, in which the probability that the photon can produce the hadronic fluctuation ($q\bar{q}$ -pair) is removed, $k_{\perp 0}$ is the transverse momentum of the quarks of the pair. In general, the eikonal function χ is expressed through the parton densities inside of the hadronic fluctuation of the photon and inside the nucleon target,

$$\chi(s, b, k_{\perp 0}^2) = A(b) \int dp_{\perp 1}^2 \int dx_1 \int dx_2 \times \times n_i(x_1, p_{\perp 1}^2, k_{\perp 0}^2) n_j^p(x_2, p_{\perp 1}^2) \frac{d\sigma_{ij}}{dp_{\perp 1}^2}. \quad (4)$$

It is customary to separate the number densities n_i into two components: a nonperturbative, vector meson dominance (VMD) component ($k_{\perp 0} < k_{\perp 0}^0 \sim 1 \text{ GeV}$) and perturbative one. The latter corresponds to relatively high masses of the hadronic system produced by the photon. Further, one can assume that, at least at not too high energies, the VMD component is dominant and perturbative component can be neglected. It

means that partonic densities (numbers of small- x gluons) inside vector mesons are larger than inside of the $q\bar{q}$ -pair with large $k_{\perp 0}$.

The well-known example of such an approach is the so-called Aspen model ("QCD-inspired eikonal model"), see [5] for a review. In this model the starting point is the eikonal $\chi(s, b)$ for the case of even scattering hadronic amplitudes ($\frac{1}{2}(f_{pp} + f_{\bar{p}\bar{p}})$) which consists of three parts:

$$\chi(s, b) = \chi_{qq}(s, b) + \chi_{qg}(s, b) + \chi_{gg}(s, b), \quad (5)$$

corresponding to quark-quark, quark-gluon and gluon-gluon interactions. In particular,

$$\chi_{gg}(s, b) = A(b, \mu_{gg})\sigma_{gg}(s). \quad (6)$$

If $f_g(x) \sim x^{-(1+\epsilon)}$ (gluon structure function), one has $\sigma_{gg} \sim s^\epsilon$.

A factor $A(b, \mu_{gg})$ is the impact parameter distribution. The parameters $\epsilon, \mu_{gg}, \dots$ are determined from experiment. The total cross section of proton-proton interaction, in the unitary (black-body) limit, is

$$\begin{aligned} \sigma_{tot}(s) &\approx 2 \int [1 - e^{-\chi_{gg}(s, b)}] d^2b = \\ &= 2\pi \left(\frac{\epsilon}{\mu_{gg}}\right)^2 \ln^2 \frac{s}{s_0} \end{aligned} \quad (7)$$

(at asymptotic energies), $A(b, \mu)|_{b \rightarrow \infty} \sim e^{-\mu b}$.

Now, using VMD and the additive quark model, the eikonal of γp -scattering can be written as [5]

$$\chi^{VMD}(s, b) = \frac{2}{3} \sigma_{gg}(s) A(b, \sqrt{3/2} \mu_{gg}). \quad (8)$$

From here one has

$$\sigma_{\gamma p} = P_{had}^{VMD} \cdot 2 \int [1 - e^{-\chi^{VMD}}] d^2b \sim \ln^2 \frac{s}{s_0}, \quad (9)$$

where P_{had}^{VMD} is the probability of the photon-vector meson transition, $P_{had}^{VMD} \sim 4\pi\alpha_{em}/f_\rho^2$.

3. Regge models

It is well known that in the Regge approach to deep inelastic scattering (DIS) the (effective) Pomeron intercept depends on Q^2 and x_{Bj} . The Q^2 -dependence can be connected with DIS dynamics (e.g., with DGLAP evolution) as well

as with the existence of an additional "hard" Pomeron. Correspondingly, there are two Regge-type parametrizations of the proton structure functions, which are frequently used.

In Donnachie-Landshoff model of two Pomerons [6] one has, in the Regge limit,

$$F_2(x_{Bj}, Q^2) = \sum_i A_i \left(\frac{Q^2}{Q^2 + a_i} \right)^{1+\epsilon_i} x_{Bj}^{-\epsilon_i}, \quad (10)$$

and, in the photoproduction limit, $\sigma_{\gamma p}$ is finite. The example of the fit is [6] ($2\nu = s[\text{GeV}^2]$):

$$\sigma_{\gamma p} = 0.283(2\nu)^{0.418} + 65.4(2\nu)^{0.0808} [\mu b]. \quad (11)$$

Authors warn against using the Eq. (10) at extremely small x_{Bj} where the fixed-power behavior will be moderated by shadowing suppression.

In the CKMT model [7] the main statement is that the rescattering (absorption) corrections in applications of the Regge theory to DIS at HERA energies are not small. The Pomeron intercept $1 + \epsilon_0 = 1.0808$ is not the true Pomeron intercept (i.e., it does not correspond to the "bare Pomeron"), but rather is the effective one. The relative contribution of the most important absorptive corrections depends on Q^2 . As a result, at large Q^2 we see the bare Pomeron (with intercept ~ 0.25) and at $Q^2 = 0$ we see the Pomeron with effective intercept 1.0808 (i.e., in the photoproduction limit, there is no term fastly growing with the photon energy).

One should mention also the Regge-type model of [8], where the concept of the effective Pomeron intercept is used, the value of which is weakly dependent, in the photoproduction limit, on the photon energy.

4. Colour dipole models

Real photons and virtual photons with small x_{Bj} have hadronic properties. The length of hadronic fluctuations is large (and larger than the target size) at large energies, $l \sim \frac{1}{m_p x_{Bj}}$ for virtual photons and $l \sim \frac{2E_\gamma}{M_{q\bar{q}}^2}$ for real photons ($M_{q\bar{q}}$ is the invariant mass of the hadronic fluctuation). So, γN -scattering is a two-step process.

The basic equations of the colour dipole model are very simple due to the fact that colour dipoles

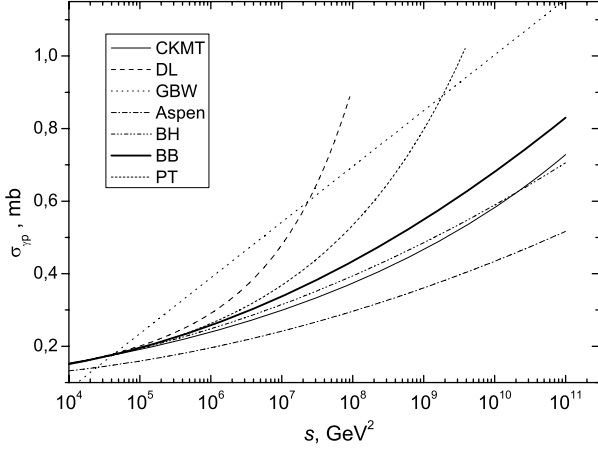


Figure 1. The photoabsorption cross section as a function of photon energy, for different models: CKMT [7], DL [6], GBW [13], Aspen [5], BH [2], BB [1], PT [8].

are eigenstates of interaction in QCD and the well known method of eigenstates [9] can be used. The total γp - cross section is expressed through the total gluon-nucleon cross section,

$$\sigma_{\gamma p} = \int d^2 r_{\perp} \Psi_{\gamma}(r_{\perp}) \hat{\sigma}(r_{\perp}). \quad (12)$$

Here, $\Psi_{\gamma}(r_{\perp})$ (more exactly, $\Psi_{T,L}(r_{\perp}, z, Q^2)$) is the photon-dipole wave function which depends on the photon virtuality, on the longitudinal momentum fraction z carried by the quark, and on the dipole size r_{\perp} . These variables are "frozen" during interaction.

The dipole cross section, $\hat{\sigma}(r_{\perp})$, is assumed to be flavour-independent and depending, except of r_{\perp} , on $s = W^2$ or x' , where x' is the momentum fraction of the proton carried by the gluon attached to the $q\bar{q}$ -loop.

Perturbative QCD leads to the formula [10]

$$\hat{\sigma}(r_{\perp}, x') = \frac{\pi^2}{3} r_{\perp}^2 \alpha_s(\bar{Q}^2) x' g(x', \bar{Q}^2), \quad (13)$$

where \bar{Q}^2 is the energy scale depending on the dipole size, $g(x', \bar{Q}^2)$ is the gluon density.

In MFGS model [11] it is assumed, phenomenologically, that $\bar{Q}^2 = \frac{\lambda}{r_{\perp}^2}$, $\lambda = \text{const}$ ($4 \div 10$).

Using unitarity, as a guide, the dipole cross section can be written as

$$\begin{aligned} \hat{\sigma}(r_{\perp}, x') &= 2 \int d^2 b [1 - e^{-\chi(b, E_{\gamma}, r_{\perp})}] \equiv \\ &\equiv 2 \int d^2 b \Gamma(E_{\gamma}, r_{\perp}, b). \end{aligned} \quad (14)$$

The profile factor $\Gamma(E_{\gamma}, r_{\perp}, b)$ is smaller than 1. If $\hat{\sigma}(r_{\perp}, x')$ is known one can calculate Γ assuming some law of t -dependence of the scattering amplitude.

Rogers and Strikman [12] calculated, using MFGS model, the γp - cross section up to super-high energies. They unitarized cross section "by force", calculating profile function Γ , and, if the profile function exceeded unity, putting $\Gamma = 1$. By such a way they determined the maximum rise of $\sigma_{\gamma p}$ with energy, $\sigma_{\gamma p} \sim \ln^3 E_{\gamma}$, for $E_{\gamma} > 10^3 \text{ GeV}$.

The dipole cross section $\hat{\sigma}(r_{\perp}, x')$ in MFGS model rises with energy infinitely. If we suppose that this cross section is bound by an energy independent value, as in simple saturation models [13], the rise of $\sigma_{\gamma p}$ with energy still will take place, due to the photon wave function. In the model of [13] one assumes that

$$\hat{\sigma}(x', r_{\perp}) = \sigma_0 [1 - e^{-\frac{r_{\perp}^2}{4R_0^2(x')}}], \quad (15)$$

$R_0^2(x') = (x'/x_0)^{\lambda} \text{ GeV}^{-2}$, $x' \sim x_{Bj}$. If the Bjorken variable, in the photoproduction limit, is modified to be

$$x = x_{Bj} (1 + \frac{4m_q^2}{Q^2}) \rightarrow \frac{4m_q^2}{W^2} \quad (Q^2 \rightarrow 0), \quad (16)$$

it is easy to show that $\sigma_{\gamma p} \sim \ln E_{\gamma}$.

The modification of the model [13], with taking into account the QCD evolution of the gluon distribution [14],

$$\hat{\sigma}(x', r_{\perp}) = \sigma_0 [1 - e^{-\frac{\pi^2 r_{\perp}^2 \alpha_s(\mu^2) x' g(x', \mu^2)}{3\sigma_0}}] \quad (17)$$

(here the scale μ^2 is assumed to have the form $\mu^2 = \frac{C}{r_{\perp}^2} + \mu_0^2$), gives, in asymptotics, $\sigma_{\gamma p} \sim \ln^{1/2} E_{\gamma}$.

Physically, the saturation model, in its phenomenological variant, corresponds to the proton

being a disc in the transverse plane with a sharp border. Saturation leads to an uniform blackening of the disk with decreasing x_{Bj} (for a parent dipole with a fixed size r_{\perp}) without changing the disc size.

For a GBW model [13] the parameters are:

$$\lambda = 0.3 ; x_0 = 3 \times 10^{-4} ; \sigma_0 = 23\text{mb}. \quad (18)$$

5. Unitary bounds on s-dependence of $\sigma_{\gamma p}$

Using three assumptions: *i*) γN -interaction is a two-step process, *ii*) generalized VDM (a dispersion relation with variable M^2 , where M is the mass of the hadronic fluctuation), *iii*) the hadronic interaction of the fluctuation ($q\bar{q}$ -pair) is a black disc interaction, one can show that the $\sigma_{\gamma^* N}$ is given by [15]

$$\sigma_{\gamma^* N} = \frac{\alpha_{em}}{3\pi} \int \frac{R(M^2)M^2 dM^2}{(Q^2 + M^2)^2} \sigma_{M^2 N}(s), \quad (19)$$

where $R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$. If we suppose that the hadronic cross section $\sigma_{M^2 N}$ is given by the black-body limit, $\sigma_{M^2 N} \sim \ln^2 \frac{s}{s_0}$, one obtains, in the photoproduction limit,

$$\sigma_{\gamma N} \sim \ln^2 \frac{s}{s_0} \ln \left. \frac{M_{max} + Q^2}{M_{min} + Q^2} \right|_{Q^2 \rightarrow 0} \sim \ln^3 \frac{s}{s_0}. \quad (20)$$

It is the so called Gribov bound [16]. However, the hypothesis of the black disk interaction cannot be correct for $q\bar{q}$ -pairs with large mass $M_{q\bar{q}}$. Such a pair has a small $r_{\perp} \sim 1/M_{q\bar{q}}$ and, being colour neutral, interacts with the target weakly, $\sigma \sim r_{\perp}^2$ (it is predicted by pQCD). Due to this, there is the following constraint on the value of M [17]:

$$M^2|_{max} \sim \Lambda^2 e^{\sqrt{a \ln(1/x_{Bj})}} \quad (21)$$

(Λ is the QCD scale), so, in far asymptotics one obtains the corrected unitary bound (assuming that $x_{Bj}^{min} \sim m_q^2/s$, as in Eq. (16)) [17]:

$$\sigma_{\gamma N} \sim \ln^2 \left(\frac{s}{s_0} \right) \ln^{1/2} \left(\frac{1}{x_{Bj}^{min}} \right) \sim \ln^{2.5} \frac{s}{s_0}. \quad (22)$$

6. Conclusions

The straggling of theoretical predictions for $\sigma_{\gamma p}$ at $E_{\gamma} \sim 10^{19} - 10^{20} eV$ is large, but not catas-

trophically large. At $s = 10^{11} \text{GeV}^2$ the predictions are in interval $(0.5 \div 1.1) \text{mb}$.

It is rather difficult to predict reliably the asymptotic s -dependence of the $\sigma_{\gamma p}$. Most probably, the upper limit on the rise of the photoabsorption cross section is given by the law $\sim \ln^3 E_{\gamma}$ as follows from the estimates of authors of [12] based on the colour dipole model. The corrected Gribov unitary bound based on generalized VDM and pQCD constraint gives $\sigma_{\gamma p} \sim \ln^{2.5} E_{\gamma}$. Predictions based on the vector meson dominance and additive quark model give the law $\sim \ln^2 E_{\gamma}$. Finally, the Regge eikonal model [7] predicts, asymptotically, $\sigma_{\gamma p} \sim E_{\gamma}^{0.1}$.

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